An Origin for Pulsar Kicks in Supernova Hydrodynamics

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It is now believed that pulsars comprise the fastest population of stars in the galaxy. With inferred mean, root-mean-square, and maximum 3-D pulsar speeds of ∼300-500 km/s, ∼500 km/s, and ∼2000 km/s, respectively, the question of the origin of such singular proper motions becomes acute. What mechanism can account for speeds that range from zero to twice the galactic escape velocity? We speculate that a major vector component of a neutron star's proper motion comes from the hydrodynamic recoil of the nascent neutron star during the supernova explosion in which it is born. Recently, theorists have shown that asymmetries and instabilities are a natural aspect of supernova dynamics. In this paper, we highlight two phenomena: 1) the "Brownian-like" stochastic motion of the core in response to the convective "boiling" of the mantle of the protoneutron star during the post-bounce, pre-explosion accretion phase, and 2) the asymmetrical bounce and explosion of an aspherically collapsing Chandrasekhar core. In principle, either phenomenon can leave the young neutron star with a speed of hundreds of kilometers per second. However, neither has yet been adequately simulated or explored. The two-dimensional radiation/hydrodynamic calculations we present here provide only crude estimates of the potential impulses due to mass motions and neutrino emissions. A comprehensive and credible investigation will require fully three-dimensional numerical simulations not yet possible. Nevertheless, we have in the asymmetric hydrodynamics of supernovae a natural means of imparting respectable kicks to neutron stars at birth, though speeds approaching 1000 km/s are still problematic.

INTRODUCTION

Recent data on pulsar proper motions (1), a recalibration of the pulsar distance scale (2), and a general recognition that previous pulsar surveys were biased towards low speeds (3) imply that many pulsars have high velocities. Mean three-dimensional galactic speeds of 450±90 km/s (3) have been estimated, with measured transverse speeds of individual pulsars ranging from zero to as much as ∼1500 km/s. Impulsive mass loss in spherically symmetric supernova explosions in binaries has long been known to impart to

the remaining neutron stars velocities that reflect orbital speeds (4,5). The magnitude of these kicks depends upon the characteristics of the binaries at explosion, which in turn depend upon mass transfer and loss and common envelope evolution before explosion. If the explosion is spherical, given the binary masses, the orbital eccentricity, and the separation at explosion, the speed of the young neutron star with respect to the system center of mass is uniquely determined.

However, Dewey and Cordes (6) performed Monte Carlo simulations of the evolution of progenitor binaries under the assumption that the explosions were spherical and were not able to reproduce the observed distribution of pulsar scintillation velocities (7). They were compelled to posit an intrinsic "kick" with a mean magnitude of $\sim$100 km/s, in addition to the speed naturally imparted due to a binary origin. Such calculations depend sensitively upon the assumed mass function, mass ratio distribution, initial semi-major axis and eccentricity distributions, and mass loss and common-envelope algorithms. Recently, Iben & Tutukov (8) have constructed a "scenario model" with state-of-the-art inputs and conclude that an "ad hoc universal kick" is not required. They claim to reconcile the presence of neutron stars in globular clusters and wide binaries and the statistics of OB runaway stars, HMXB's, LMXB's, and radio pulsars with spherical explosions. However, the pulsar speed distribution they derive has a mean near 100–150 km/s, marginally consistent only with the old pulsar distance scale (2), ignoring the bias of pulsar searches to the plane (3). They are able to produce maximum speeds near 1000 km/s from the explosion of a $\sim$16 $M_\odot$ helium core in a tight binary, but are unable to reproduce the large fraction of pulsars with speeds above 500 km/s now inferred (reference 3 and J. Cordes, private communication).

There has long been indirect evidence that neutron stars are given a kick at birth. Burrows & Woosley (9) concluded that if the progenitor system of PSR B1913+16 underwent common envelope evolution and the pre-explosion orbital eccentricity was zero, the current observed neutron star masses and orbit parameters are inconsistent with both a spherical explosion and known helium core radii. The pre-explosion Roche limit would have been well within the progenitor helium core before the explosion and spiral-in would have been unavoidable. They concluded that an intrinsic kick of at least 170 km/s was indicated. The same analysis with the same conclusions can be done for PSR B1534+12. (However, if there were no common envelope phase and the pre-explosion eccentricity were not zero, an intrinsic kick would not be required (10).) Recently, Wasserman, Cordes, and Chernoff (this conference) have shown that in PSR B1913+16 the observed misalignment of the pulsar spin axis with the orbit axis is best explained if the pulsar received an intrinsic kick of as much as $\sim$800 km/s. Furthermore, van den Heuvel & Rappaport (11) have argued that the large eccentricities of Be/Neutron star binaries imply the existence of intrinsic kicks.

That most supernova remnants did not seem to have neutron stars, neither radio pulsars nor point X-ray sources, within them has long encouraged spec-

ulation concerning the yield of black holes in supernova explosions (12,13). However, Caraveo (14) and Frail, Goss & Whiteoak (15) have recently identified pulsars with young SNR's in a majority of SNR's with putative ages less than 20,000 years. These identifications depend only on high relative transverse velocities with an inferred average value of ~500 km/s. Excitement with these new associations must be tempered by at least three caveats. First, the actual proper motions of these pulsars have yet to be measured. Second, the centroid of the SNR reflects in part the mass distribution of the ISM into which the supernova exploded and may not coincide with the position of the actual explosion. Third, when a supernova explodes in a binary, the kick due to orbital motion is in the opposite direction to the recoil of the ejecta. The inferred pulsar transverse speed is actually the relative speed between the debris and the pulsar which could be a factor of one and a half or two times larger than the pulsar's speed with respect to the center of mass. Therefore, the large inferred speed may not indicate a large intrinsic kick, but a modest orbital kick in a spherical explosion.

Related to the emerging SNR/pulsar associations are the new data on the "systemic" velocities of young supernova remnants (16,17) (with ages from only hundreds to a few thousand years). It is observed that the "center of mass" velocity of oxygen clumps in these explosions is different from that of the local ISM by -500 km/s, +900 km/s, +370 km/s, and -500 km/s for N132D, Cas A, SN0540-69.3, and E0102.2-7219, respectively (17). These data imply that the explosion itself was asymmetric, but asymmetries in the ISM and a high progenitor speed with respect to the ISM can not yet be ruled out. It is intriguing to speculate that the vector velocity of the neutron star or black hole residue of these explosions could be (anti)correlated with such systemic velocities. Nevertheless, there are many other indications that supernova explosions are aspherical and asymmetrical. Utrobin *et al.* (18) interpret the "Bochum" event in Hα in SN1987A with a 10^{-3} $M_\odot$ shard of ^{56}Ni moving at ~4700 km/s. The jagged optical and IR line profiles of SN1987A, the intrinsic polarization of spectral features in SN1993J and SN1987A, the oblateness of recent HST images of SN1987A, and the shrapnel observed in the Vela SNR (19), all hint that the explosions are asymmetrical. Such asymmetries may have counterparts in correlated neutron star recoils.

PREVIOUS THEORIES FOR INTRINSIC PULSAR KICKS

Theories concerning the origin of intrinsic neutron star kicks are few and generally undeveloped. They can be divided into those that rely primarily on some aspect of the neutron star's magnetic field and those related to the supernova's dynamics or neutrino emissions. The former class are in part motivated by the suggestion that pulsar proper motions are correlated with their measured magnetic dipole moments (20). Data in support of this hypothesis are no longer compelling, particularly when selection biases are acknowledged

(1). Nevertheless, the proposed magnetic models have been clever and are worthy of review. Harrison & Tademaru (21) suggested that the magnetic dipole of a neutron star could be off-center. If the star rotated, not only would the canonical magnetic dipole radiation be emitted, but a net linear momentum would be radiated. The neutron star would recoil and it could be accelerated for hundreds of years. This theory predicted a correlation between the pulsar's spin axis and its proper motion that some do not find (20) and relies on short birth spin periods to achieve high speeds.

Chugai (22) noted that if the neutrino had a magnetic moment (and, hence, a mass), the interaction of the emerging neutrinos with a strong young pulsar field could lead to anisotropic neutrino emission that could accelerate the neutron star. However, fields above 10^{14} gauss were required to provide a useful kick and it is known that the dipole fields of the fastest pulsars are "unexceptional." A variation on this model is provided by Bisnovatnyi-Kogan (this volume), who relies upon the strong and anisotropic magnetic field to alter the opacity of the protoneutron star matter from which the emitted neutrinos decouple.

Woosley (23) published that if the integrated emission of $\sim 3\times 10^{53}$ ergs in neutrinos were radiated with an anisotropy of but 1%, recoil speeds of $\sim$300 km/s were possible. This simple fact highlights the great potential the 0.15 $M_\odot$ of relativistic neutrinos has to accelerate the nascent neutron star. However, these neutrinos are radiated over many seconds and it is not clear what processes can maintain an emission asymmetry such that a *net* asymmetry of percents survives. Note that during the first seconds in the life of the protoneutron star, only about one half to one third of the neutrino energy is radiated. It may be during this early phase that "convective" motions can produce neutrino hot spots, but these hot spots wander and the magnitude of the net emission asymmetries during early vigorous convection is unclear. We have recently calculated this effect, but not definitively, and we present our results in the next section. It could be that rapid rotation plus convective overturn interact to give respectable neutrino recoils. Shimizu, Yamada, & Sato (24) have recently estimated the magnitude of rotation-induced neutrino emission anisotropies, but too much concerning this effect remains unresolved to draw clear conclusions.

Instabilities in the protoneutron star prior to and during the supernova explosion have recently been implicated in supernova explosions (25–27). This led Herant, Benz, & Colgate (28) to posit a cascade to an "$\ell = 1$" mode as the supernova develops. Such a dipole mode in the hydrodynamics would imply a neutron star recoil. However, Burrows, Hayes, & Fryxell (25) see no such cascade to lower-order modes, and Khokhlov (29) has identified such a cascade as an artifact of 2-D hydrodynamics.

RECOILS DUE TO HYDRODYNAMIC MOTIONS AND NEUTRINO ANISOTROPIES

The velocity a neutron star ends up with is a vector sum of many contributions: the galactic rotation, the progenitor's motion with respect to the galactic rotation, the orbital kick (or kicks, if there were two supernovae in the binary), the deceleration due to motion in the galactic potential, and any intrinsic kicks imparted. It is the perceived necessity of intrinsic kicks that motivates this work.

Spherical explosions do not kick the residual neutron star. However, asymmetries in the collapse and before and after the reignition of the supernova have the potential to impart to the core respectable recoils via either mass motions or anisotropic neutrino emissions. The latter could be a consequence of anisotropic radial opacity profiles or anisotropic accretion luminosities. That core-collapse supernovae are subject to Rayleigh-Taylor instabilities has been independently demonstrated by at least three theoretical groups (25–27). The pre-explosive core experiences a convective "boiling" phase behind the temporarily stalled shock and the explosion, when it occurs, erupts in bubbles, fingers, and plumes. To date, these hydrodynamic calculations have been performed in only two dimensions (with axial symmetry) and with a variety of simplifying approximations. Only three-dimensional calculations with much-improved neutrino transfer and realistic 3-D progenitor cores that incorporate rotation and the hydrodynamics of pre-collapse convective burning can adequately address the issue of the kick imparted to the residue. Nevertheless, this class of "simpler" 2-D simulations can help theorists explore the potential role of hydrodynamics and neutrinos in imparting proper motions. The observed pulsar speeds of $\sim$500 km/s are only a few percent of the speeds ($\sim$30,000 km/s) achieved during the pre-explosive convective phase and during the explosive phase. In this context, the observed high pulsar speeds seem low and possible.

The **star** calculation highlighted in Burrows, Hayes, & Fryxell (25) simulates in two dimensions the spherical collapse of the core of a 15 $M_\odot$ progenitor (30), the early "mantle" convection and accretion phase (during which the shock pauses), and the subseqent explosion. Asymmetries, Rayleigh-Taylor instabilities, and overturning motions abound in this protoneutron star. The character of this dynamics is described in (25) and in Burrows & Hayes (31) and will not be detailed here. This calculation is not best-suited to explore recoils, since it is in 2-D, the central core ($<$ 15 km) is fixed and is treated in 1-D, and only a 90° wedge (with circular symmetry) from 45° to 135° in the spherical coordinate, θ, is followed (the calculation is "capless."). Nevertheless, around the core the accretion rate per area is fluctuating wildly and the pressure field is aspherical. One can calculate the total momentum of all the matter exterior to the core and, using momentum conservation, infer the impulse to the core. Note that since periodic boundary conditions are used in the angular direction and the core is fixed (and, hence, can absorb momen-

FIG. 1. The inferred velocity (in km/s) in the polar (z) direction with which the core recoils in response to the overturning motions in the **star** calculation of Burrows, Hayes, & Fryxell (25). Note that the cap from θ equals 0° to 45° is missing in this calculations.

tum), the momentum of the outer material from 15 to 4500 kilometers need not be zero. (This is a subtle point.) Shown in Figure 1 is the inferred core recoil velocity in the polar (z) direction due only to the wedge's fluctuating motions exterior to the core. The corresponding value in the "x" direction is zero due to azimuthal symmetry.

Figure 1 depicts two components, one that fluctuates on overturn timescales of 3–5 milliseconds, and a longer-term component that first grows to 100–180 km/s and then subsides to near zero. The timescale of the latter is the supernova delay timescale of $\sim$100 milliseconds to peak. The fluctuating component represents the "Brownian" shaking that the core must experience during the pre-explosive boiling and early explosion phases. The secular term grows, then shrinks because the impulse to the core is the product of a "mass" with a "velocity." The mass between the shock and the core's neutrinosphere steadily decreases with time during the delay phase, while the vigor (speed) of the overturning and fluctuating motions is steadily increasing. The product peaks and then decreases. In this simulation, the explosion happened near t = 309 milliseconds (just after peak). (In reality, the core's recoil need not subside much nor asymptote to zero, but should decouple in a way that it can

not in this calculation because of the constraints.) While the **star** calculation is inadequate to truly explore neutron star recoils, it suggests that 3-D speeds of hundreds of kilometers per second might in fact be imparted during the pre-supernova and supernova phases by hydrodynamic motions. The impulse due to asymmetric neutrino radiation was included in this calculation and by its end (t = 410 milliseconds) the neutrino contribution to the inferred recoil was about $\sim$20%. It is interesting to point out that the magnitude of the peak in Figure 1 depends on the product of the "convecting" mass and the overturn speeds (and the delay to explosion), which in turn depend upon the density structure of the progenitor. A denser core from a more massive ZAMS star may achieve a higher peak recoil in the sense described above (ceteris paribus) and there may be an interesting relationship between a pulsar's proper motion and its progenitor mass. More realistic calculations are clearly needed to investigate this (32).

The **star** calculation of Figure 1 was done assuming that the collapsing "Chandrasekhar" core and implosion were spherical. Recent hydrodynamic calculations (33) of convection during the shell oxygen and silicon burning that immediately precedes core collapse in massive stars suggest that asymmetries at collapse in density, velocity, and composition can be larger than a simple mixing-length prescription would imply. Furthermore, rotation might interact with convection to further distort the core (34). The upshot might be an asymmetrical, aspherical collapse. If the amplitudes of the asymmetries in density or velocity are large ($\sim$percents) and if a significant low-order mode ($\ell = 1$) exists at the onset of collapse, the consequences for a pulsar's recoil can be significant. To explore this hypothesis, in January of 1995 we conducted a "toy" calculation of aspherical collapse. In this **kick** simulation, we artificially decreased by 15% the density of the Chandrasekhar core exterior to 0.9 $M_\odot$ and within 20° of the pole. This calculation was done in 2-D with azimuthal symmetry, but θ ranged between 0° and 180°. Hence, the entire core, not just a wedge, was followed. A 15% decrease in density is larger than yet seen in the calculations of Bazan & Arnett (33), but we imposed no initial aspherical perturbation in velocity, despite the up to Mach 0.25 asymmetries they have derived. The essential point is that initial asphericities in the **kick** run grew during collapse, so that the mass column depths in various angular directions diverged. The matter collapsed at different rates in different directions, though pressure forces were transmitted in the angular directions as well that partially smoothed the deviations. Figure 2 depicts the core early in the collapse. The bounce was delayed on the side of the perturbation wedge and the resulting shock bowed out in the wedge direction. The accretion rates through this shock were highly aspherical. To avoid burning CPU in what was merely a toy "proof-of-principle" calculation, we artifically hardened the emergent neutrino spectrum to facilitate an early explosion. The electron-type spectra were assumed to have "η's" of 3, above the 1.5–2.0 normally encountered in fits to more realistic spectra (35). Since neutrino heating drives supernovae, this ignited the explosion within 10 milliseconds of bounce. The subsequent

FIG. 2. A grey-scale rendition of the entropy distribution early in the collapse of the core constructed for the **kick** simulation. Velocity vectors are superposed. Note the wedge cut out near the pole at the left. The physical scale is 2000 km from the center to the edge. Darker color indicates lower entropy.

explosion was aspherical not only due to the normal instabilities, but also due to the asphericity of the matter into which the explosion emerged and/or was driven.

Figure 3 depicts the flow late in the explosion. The explosion erupted preferentially through the path of least resistance, *i.e.* in the direction of the wedge that we had imposed. In the **kick** simulation, this wedge collapsed more slowly than the rest of the core. Since neutrino heating drives the explosion, matter heated near the neutrinosphere expanded out as if from a reaction chamber. The protoneutron star residue received a significant impulse à la the rocket effect. Furthermore, the core bounced asymmetrically and even without the neutrinos the central residue would have recoiled away from the direction of retarded collapse. (It is possible that the purely hydrodynamic effect is

FIG. 3. A grey-scale rendering of the entropy distribution at the end of the **kick** simulation, about 50 milliseconds into the explosion. Note the pronounced left-right asymmetry in the ejecta and the velocity field (as depicted with the velocity vectors). The lengths of the velocity vectors in Figures 2 and 3 have different scalings. The physical scale is 2000 km from the center to the edge. Darker color indicates lower entropy and the $\theta = 0$ axis points to the left.

FIG. 4. The impulse (in *cgs*) imparted to the core versus time (in seconds) in the **kick** simulation. The initial momentum is approximately zero, but grows systematically after bounce in the direction opposite to the artificial wedge, cut into the core to mimic an asymmetry just before collapse. Shown are the impulses due to neutrino asymmetry (dashed), mass motions (dotted), and the sum (solid).

larger than the rocket effect, but we have yet to explore either satisfactorily.) The impulse on the core versus time and the inferred recoil speed are depicted in Figures 4 and 5. An asymmetry in collapse translates into a clear kick, though the initial total momentum be almost zero. Initially, the recoil is in the direction of the wedge since the rest of the matter bounced first. Afterwards, as the shocked matter starts to squirt through the region of least resistance and column depth, the recoil changes sign and grows inexorably to its asymptotic value. Figure 4 shows that the recoil speed in the **kick** simulation reached $\sim$500–600 km/s. This is large, but only 2% of the speed of the supernova ejecta. The contribution of neutrino radiation asymmetry to this kick is also depicted in Figures 4 and 5 and amounts to $\sim$20%. It is in the same direction as the mass motion effect, due to the fact that fluxes are larger on the thin, or low-column, side. Whether this is generically true remains to be seen.

The major conclusion of the toy **kick** simulation is that initial collapse and core asymmetry can translate into an appreciable neutron star recoil due to the variation in the collapse time with angle, the asymmetrical bounce, the variation in the tamp with angle, and the rocket effect. Quite naturally,

FIG. 5. Same as Figure 4, but the inferred recoil speed (in km/s) of the residual neutron star versus time (in seconds).

the debris and the residue move in opposite directions, conserving total momentum. Since accretion-induced collapse is not preceded by the convective burning stages characteristic of the final hours of the core of a massive star, the initial asymmetries in the two contexts may be quite different. Consequently, the proper motions of AIC neutron stars and those of neutron stars from massive stars may be systematically different, with those of the latter being on average higher. However, to explore these phenomena in more detail and more credibly will require a realistically aspherical initial core, 3-D simulations, and better neutrino transfer. Nevertheless, it is heartening that proper motions of the "correct" magnitude are produced in these embryonic simulations of multi-dimensional supernovae.

Given the stochastic nature of the processes we have highlighted here by which we have attempted to explain intrinsic pulsar kicks, we expect that Nature provides not a single high kick speed, but a broad distribution of speeds. These will depend upon the degree and character of the initial asymmetries, the initial rotation structure, the duration of the delay to explosion, the progenitor density profiles, and chance.

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REFERENCES

1. P.A. Harrison, A.G. Lyne, & B. Anderson, Mon. Not. R. Astron. Soc. **261**, 113 (1993).
2. J.H. Taylor & J.M. Cordes, Astrophys. J., **411**, 674 (1993).
3. A. Lyne & D.R. Lorimer, Nature **369**, 127 (1994).
4. J.R. Gott, J.E. Gunn, & J.P. Ostriker, Astrophys. J. **160**, L91 (1970).
5. V. Radhakrishnan & C.S. Shukre, in *Supernovae, Progenitors, and Remnants*, eds. G. Srinivasan & V. Radhakrishnan (Indian Academy of Sciences: Bangalore), pp.155–169 (1985).
6. R.J. Dewey & J.M. Cordes, Astrophys. J. **321**, 780 (1987).
7. J.M. Cordes, Astrophys. J. **311**, 183 (1986).
8. I. Iben & A. Tutukov, submitted to Astrophys. J., (1995).
9. A. Burrows & S.E. Woosley, Astrophys. J. **308**, 680 (1986).
10. G.E. Brown & J.C. Weingartner, Astrophys. J. **436**, 843 (1995).
11. E.P.J. van den Heuvel & S. Rappaport, in *I.A.U. Colloquium 92*, eds. A. Slettebak & T.D. Snow (Cambridge Univ. Press), pp. XXX (1986).
12. D.J. Helfand & R.H. Becker, Nature **307**, 215 (1984).
13. G.E. Brown & H.A. Bethe, Astrophys. J. **423**, 659 (1994).
14. P. Caraveo, Astrophys. J. **415**, L111 (1993).
15. D.A. Frail, W.M. Goss, & J.B.Z. Whiteoak, Astrophys. J. **437**, 781 (1994).
16. R.P. Kirchner, J.A. Morse, P.F. Winkler, & J.P. Blair, Astrophys. J. **342**, 260 (1989).
17. J.A. Morse, P.F. Winkler, & R.P. Kirchner, submitted to Astrophys. J. (1995).
18. V.P. Utrobin, N.N. Chugai, & A.A. Andronova, submitted to Astron. & Astrophys. (1994).
19. B. Aschenbach, R.J. Egger, & J. Trumper, Nature **373**, 587 (1995).
20. A.G. Lyne, B. Anderson, & M.J. Salter, Mon. Not. R. Astron. Soc. **201**, 503 (1982).
21. E.R. Harrison & E. Tademaru, Astrophys. J. **201**, 447 (1975).
22. N.N. Chugai, Sov. Astron. Lett. **10**, 87 (1984).
23. S.E. Woosley, in *The Origin and Evolution of Neutron Stars*, eds. D.J. Helfand & J.-H. Huang (D. Reidel: Dordrecht), p. 255 (1987).
24. T. Shimizu, S. Yamada, & K. Sato, Astrophys. J. **434**, 268 (1994).
25. A. Burrows, J. Hayes, & B.A. Fryxell, Astrophys. J., in press (1995).
26. M. Herant, W. Benz, J. Hix, C. Fryer, & S.A. Colgate, Astrophys, J. **435**, 339 (1994).
27. H.-T. Janka & E. Müller, Astron. & Astrophys., in press (1995).
28. M. Herant, W. Benz, & S.A. Colgate, Astrophys. J. **395**, 642 (1992).
29. A. Khokhlov, Astron. & Astrophys. **290**, 496 (1991).
30. T. Weaver & S.E. Woosley, Astrophys. J. Suppl., in press (1995).

31. A. Burrows & J. Hayes, to appear in the proceedings of the *17'th Texas Symposium on Relativistic Astrophysics*, held in Munich, Germany, December 12-16, 1994 in the Annals of the New York Academy of Sciences, ed. H. Boeringer.
32. H.-T. Janka & E. Müller, Astron. & Astrophys. , (1994).
33. G. Bazan & D. Arnett, Astrophys. J. **433**, 41 (1994).
34. S.A. Colgate & P.J.T. Leonard, in *Gamma-Ray Bursts*, eds. G.J. Fishman, J.J. Brainerd, & K. Hurley (A.I.P. Press), p. 581 (1994).
35. E. Myra & A. Burrows, Astrophys. J. **364**, 222 (1990).